NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 7
COURSE CODE: CAN702S	**COURSE NAME:** COMPLEX ANALYSIS
SESSION: JANUARY 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	PROF. G. HEIMBECK
MODERATOR:	PROF. F. MASSAMBA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 4 PAGES (Including this front page)

Question 1

a) Let $u \in \mathbb{C} - \{1\}$ such that $|u| = 1$. Put u into modulus-argument form and find $|1+u|^2$. Conclude that $|1+u| < 2$. [5]

b) Let $a, b \in \mathbb{C}$ such that $a \neq b$. Let $K > 0$ such that $|a|, |b| \leq K$.

   i) If $|a| < K$ or $|b| < K$, verify that $|a+b| < 2K$. [4]

   ii) If $|a| = K$ and $|b| = K$, show that $|a+b| < 2K$. [7]

Question 2

Let $S := \{z \in \mathbb{C} |\ |z| = 1\}$. Let $a \in \mathbb{C} - S$, and let $f : S \to \mathbb{C}$ be defined by

$$f(z) := \frac{a - z}{1 - \bar{a}z}.$$

a) Find $\overline{f(z)}$ and conclude that $f(z) \in S$. [5]

b) Determine $f \circ f$. [5]

c) If $|a| < 1$, prove that f has no fixed points. [7]

Question 3

a) Prove that $\mathbb{R}$ is a closed subset of $\mathbb{C}$. Why is the limit of a convergent sequence of real numbers a real number? Explain! [6]

b) Let $(a_n)_{\mathbb{N}}$ be a convergent sequence of complex numbers. Let $\varepsilon > 0$ such that $|a_n| \leq \varepsilon$ for all $n \in \mathbb{N}$. Show that $|\lim_{n\to\infty} a_n| \leq \varepsilon$. [3]

c) Let $(F_n)_{\mathbb{N}}$ be a descending sequence of non-empty, bounded and closed subsets of $\mathbb{C}$. Prove that

$$\bigcap_{n=1}^{\infty} F_n \neq \emptyset.$$

[9]

Question 4

a) Let $X \subset \mathbb{C}$, and let $f : X \to \mathbb{C}$ be a function and $a \in X$. When is f differentiable at a? State the definition. What is the derivative f' of f? [4]

b) i) Is the function $\mathrm{Im} : \mathbb{C} \to \mathbb{C}$ real-differentiable? Give reasons. [4]

ii) Show that $\mathrm{Im} : \mathbb{C} \to \mathbb{C}$ is nowhere differentiable. [4]

Question 5

Let $a, b \in \mathbb{R}$ such that $a < b$ and let $f : [a, b] \to \mathbb{C}$ be a continuous function.

a) What is meant by $\mathrm{Re}\, f$? State the definition. Further, show that $\mathrm{Re}\, f$ is a continuous function. [3]

b) State the definition of $\int_a^b f\, dt$. Why does this definition make sense? [3]

c) Prove that

$$\left| \int_a^b f\, dt \right| \leq (b-a) \max\{|f(t)| \mid t \in [a, b]\}$$

[3]

Question 6

a) Let $\sum a_k$ and $\sum b_k$ be series of complex numbers. The summation starts at 0. State the definition of the Cauchy product of these two series. Is the Cauchy product of two convergent series convergent? State what one knows in this regard. [6]

b) i) State the definition of the exponential function and verify that the power series which is used in this context, is everywhere convergent. [5]

ii) Derive the addition theorem of the exponential function and show that the addition theorem implies that the exponential function has no zeroes. [5]

Question 7

a) What is a star-shaped region? State the definition. [3]

b) State Cauchy's integral theorem for star-shaped regions. [3]

c) Let $H := \mathbb{C} - (-\infty, 0]$. Show that H is open and star-shaped with respect to 1. Conclude that the function $z \in H \to \frac{1}{z} \in \mathbb{C}$ has an antiderivative. [6]

End of the question paper.